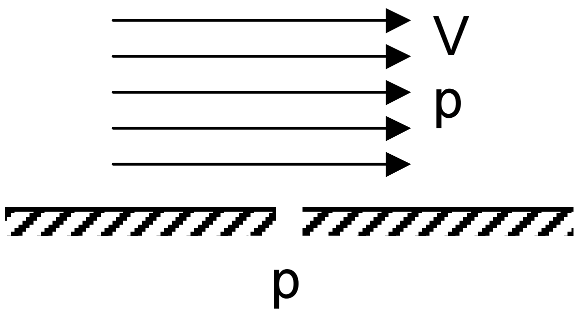
Incompressible flow (page 60):

Bernoulli's equation (steady, inviscid, incompressible):

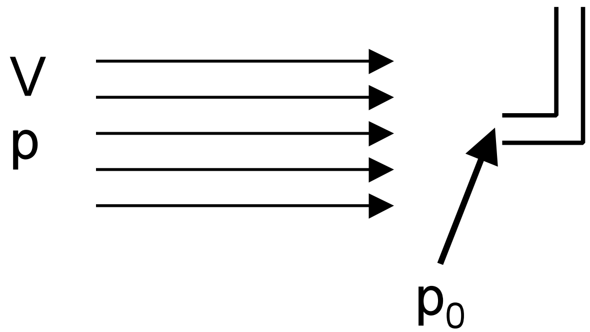
$$p + \tfrac{1}{2}\rho V^2 = p_0$$

p_0 is the stagnation (or total) pressure, constant along a streamline.

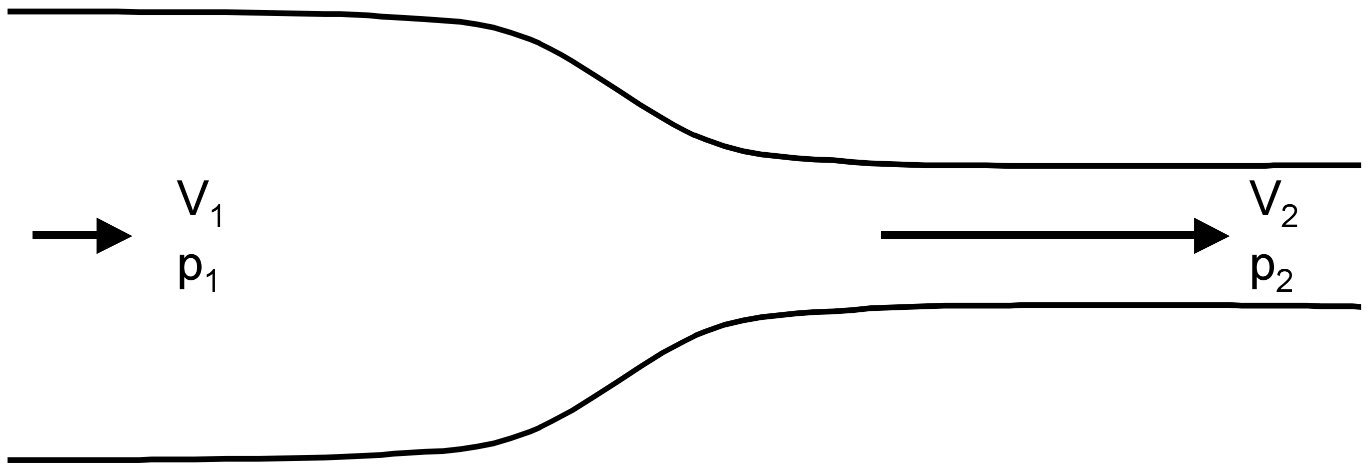
Pressure tapping in a wall parallel to the flow records static pressure



Pitot tube records the stagnation pressure (flow is brought isentropically to rest).



Why is the concept of Stagnation Pressure p_0 useful in incompressible flow?



$p_1 + \frac{1}{2}\rho V_1^2 = p_{01}$ $\qquad$ $p_2 + \frac{1}{2}\rho V_2^2 = p_{02}$

For steady, inviscid incompressible flow:

$$p_{01} = p_{02}$$

Although the velocity and pressure have changed, for steady, inviscid, incompressible flow, the stagnation pressure has the same value at points "1" and "2".

Stagnation enthalpy (page 61):

The steady flow energy equation:

$$\dot{q} - \dot{w}_x = (h_2 + \tfrac{1}{2}V_2^2) - (h_1 + \tfrac{1}{2}V_1^2)$$

Define the stagnation (or total) enthalpy as:

$$h_0 = h + \tfrac{1}{2}V^2$$

SFEE becomes:

$$\dot{q} - \dot{w}_x = h_{02} - h_{01}$$

Thus, the stagnation enthalpy only changes when heat or shaft-work are interchanged (it is independent of the local flow velocity, but does depend on the frame of reference).

The SFEE is **TRUE FOR COMPRESSIBLE AND INCOMPRESSIBLE** flow.

Stagnation temperature (page 61):

For a perfect gas, define the stagnation (or total) temperature T_0 as:

$$C_p T_0 = C_p T + \tfrac{1}{2} V^2$$

SFEE becomes:

$$\dot{q} - \dot{w}_x = C_p T_{02} - C_p T_{01}$$

Stagnation temperature only changes when heat or shaft-work are interchanged.

(It is independent of the local flow velocity, but does depend on the frame of reference).

This is why, in earlier courses, we have often ignored the KE of the flow in the SFEE.

We have actually been using stagnation temperature:

$$T_0 = T + \frac{V^2}{2C_p}$$

T_0 is the temperature of the air when brought to rest adiabatically.

Stagnation Pressure (page 62):

It is possible to define a useful reference pressure, the stagnation (or total) pressure p_0 by considering an **isentropic** deceleration between:

	physical (static) state with velocity V specified by:	T and p
and	reference stagnation state at zero velocity specified by:	T_0 and p_0

Thus:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

Note that for steady flow along a streamline:

Stagnation temperature is constant provided there is no heat or work transfer.

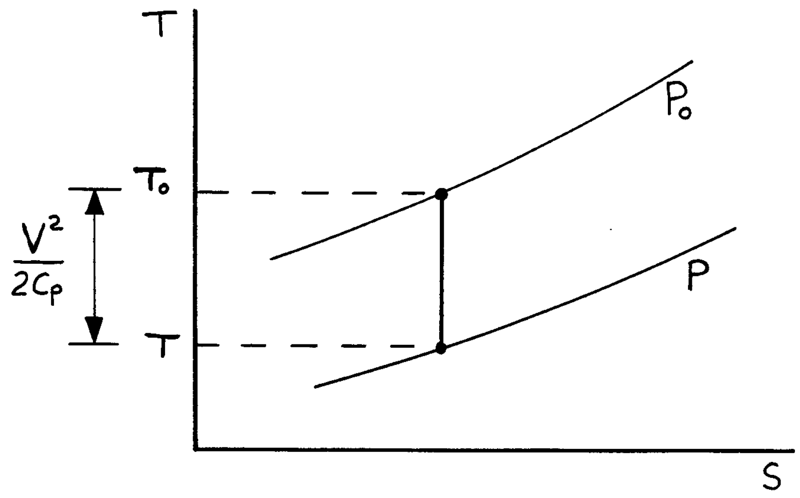
Stagnation pressure is only constant if, in addition, the flow is isentropic.

Stagnation conditions and the T-S diagram:

Stagnation conditions are easily represented on a T-S diagram since:

$$T_0 = T + \frac{V^2}{2C_p}$$

and the stagnation pressure p_0 is at the same entropy as the static conditions (T & p)



In carrying out the cycle analysis, for example finding the work or heat transfer, in a gas turbine, it is the **stagnation** quantities which should be used.

Important: Stagnation conditions depend on the frame of reference.

Why is Mach number, $M = V / \sqrt{\gamma RT}$, so important?

Stagnation temperature can be rearranged:

$$\frac{T_0}{T} = 1 + \frac{V^2}{2C_pT} = 1 + \frac{\gamma - 1}{2}\frac{V^2}{\gamma RT} = 1 + \frac{\gamma - 1}{2}M^2$$

Thus the relationship between stagnation temperature T_0 and (static) temperature T is purely a function on Mach number (and γ).

Similarly:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{V^2}{2C_pT}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

The graphs of T_0/T and p_0/p are in Thermo-Fluids Data Book (2004 edition).

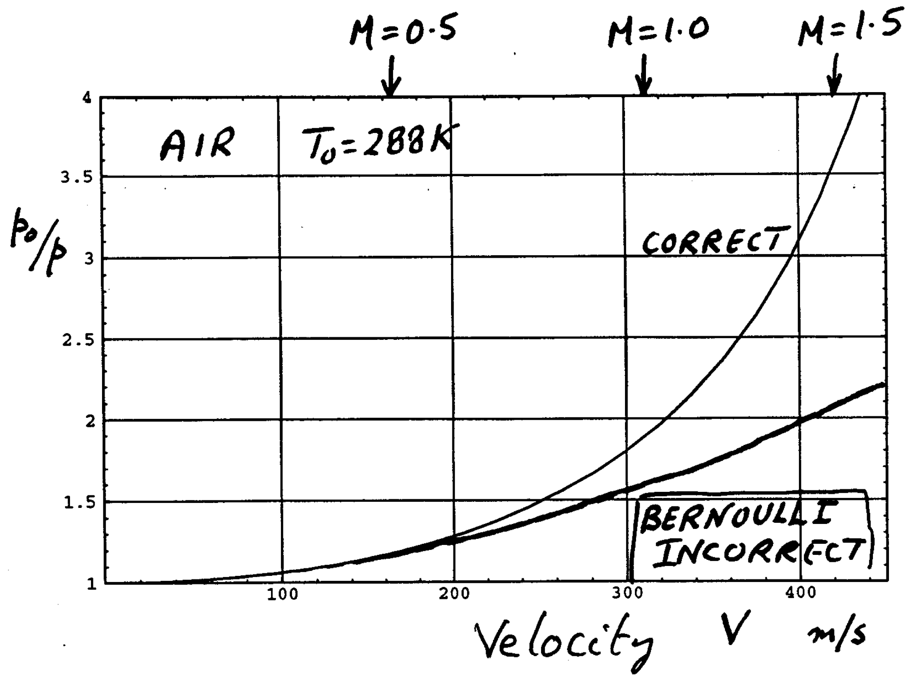
Incompressible and compressible flow (page 61):

For incompressible flow:

$$V^2 = \frac{2(p_0 - p)}{\rho}$$

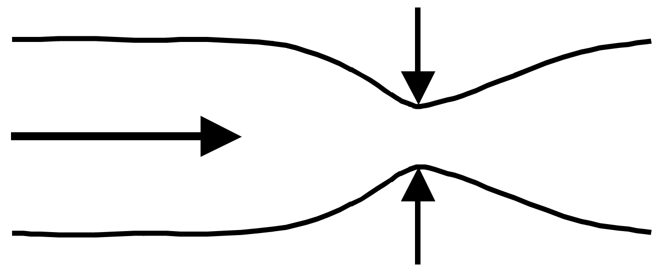
For compressible flow:

$$M^2 = \frac{2}{(\gamma - 1)}\left(\left[p_0 / p\right]^{(\gamma-1)/\gamma} - 1\right)$$



Bernoulli's Equation is **NOT** correct for the flow of gases. It should only be used where the Mach number is less than 0.3, i.e. where the effects of compressibility are small.

The choked nozzle (page 64):



The mass flow rate per unit area:

$$\dot{m}/A = \rho V$$

Maximum value of $\dot{m}/A$ occurs at the minimum area, usually called the "throat".

For steady adiabatic flow: $V = \sqrt{2C_p(T_0 - T)}$

For isentropic flow: $\rho = \rho_0 \left(\frac{T}{T_0}\right)^{1/(\gamma-1)}$

Hence: $\dot{m}/A = \sqrt{2C_p(T_0 - T)} \times \rho_0 \left(\frac{T}{T_0}\right)^{1/(\gamma-1)}$ = function(T)

The maximum mass flow per unit area can be found (by differentiation) to be when:

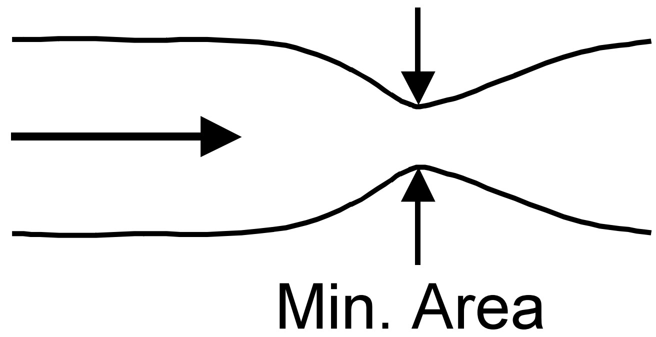
$T = 2\,T_0/(\gamma+1)$ corresponds to $V = \sqrt{\gamma RT} = a$ **ie when the Mach = 1.0**

Choked nozzles (page 64):

As the (static) pressure at the exit of a nozzle is decreased, the flow velocity will increase until the velocity is sonic at the throat (minimum area). This corresponds to the maximum possible mass flow rate through the nozzle (for fixed inlet stagnation conditions) and the nozzle is said to be choked. Lowering the exit static pressure further will not increase the mass flow rate through the nozzle.

To accelerate a flow beyond sonic velocity it is first necessary to reduce the flow area to a throat and then increase the flow area downstream of the throat.

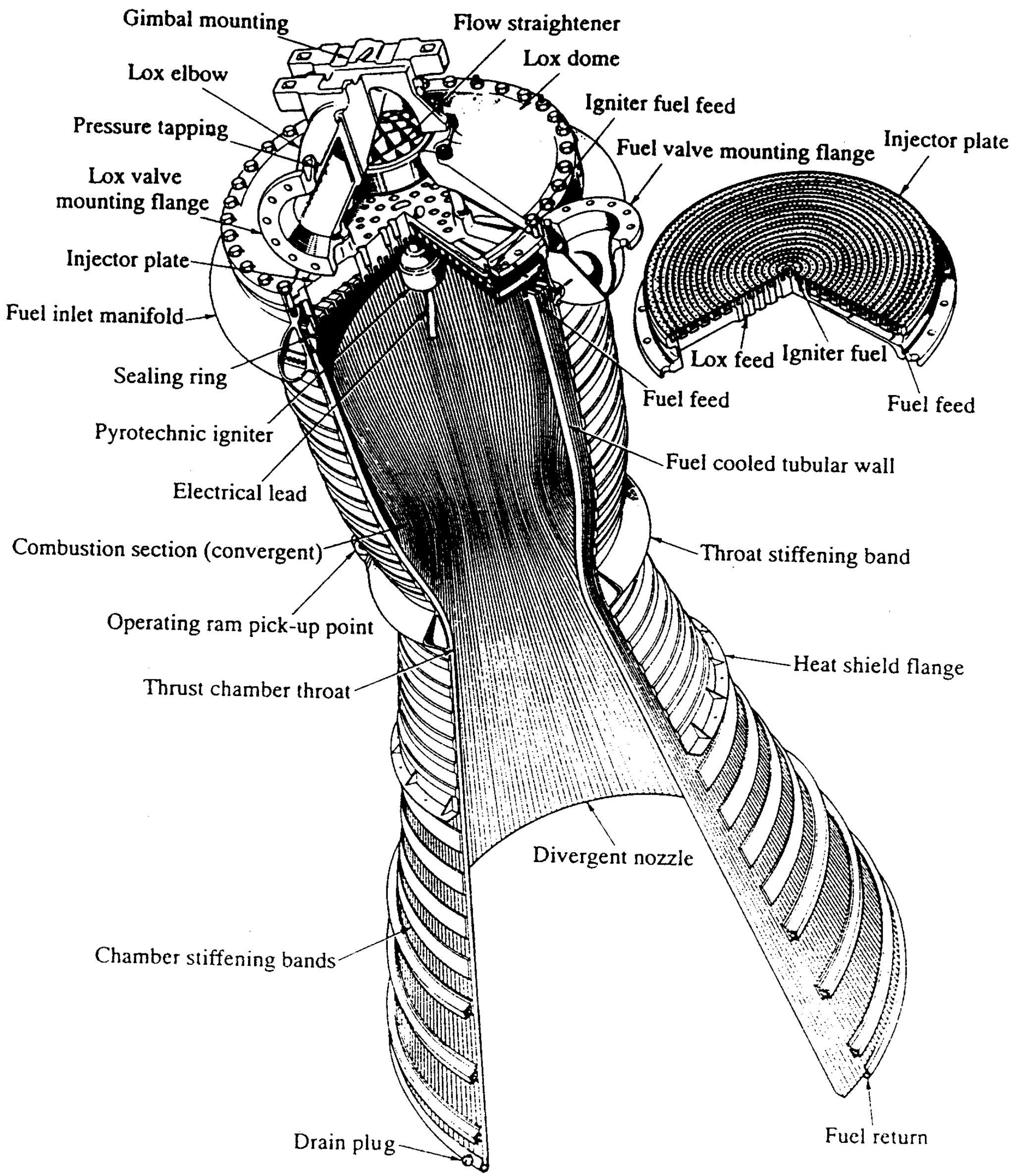
convergent-divergent (con-di) nozzle



It is not normally worthwhile fitting a con-di nozzle to a subsonic civil aircraft as the exit jet velocity will only be slightly supersonic. (Unconstrained expansion.)

Convergent-Divergent nozzles are very common in rocket motors for spacecraft.

In such applications the pressure at the nozzle exit is very low (zero?) and so the flow accelerates to a very high Mach number.



Non-dimensional mass flow function (page 65):

$\dot{m}/A$ can be expressed in non-dimensional form as:

$$\frac{\dot{m}\sqrt{C_p T_0}}{A\,p_0} = \frac{\gamma M}{\sqrt{\gamma - 1}}\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-(\gamma+1)/2(\gamma-1)}$$

This is purely a function of Mach number (and adiabatic index γ) and its graph is shown in the Thermo-Fluids Data Book (2004 edition).

At the throat, when M = 1, for the flow of air (γ = 1.4):

$$\frac{\dot{m}\sqrt{C_p T_0}}{A\,p_0} = 1.281$$

The mass flow through a choked nozzle is proportional to the inlet stagnation pressure and inversely proportional to the square root of the inlet stagnation temperature.

One-dimensional flow of a perfect gas (page 66):

The quantities:

$$\frac{\dot{m}\sqrt{C_p T_0}}{A p_0}, \quad \frac{p_0}{p}, \quad \frac{T_0}{T} \quad \text{and} \quad \frac{\rho_0}{\rho}$$

are commonly used in gas turbine calculations.

For air ($\gamma = 1.4$) the graph is in the Thermo-Fluids Data Book (2004 edition).

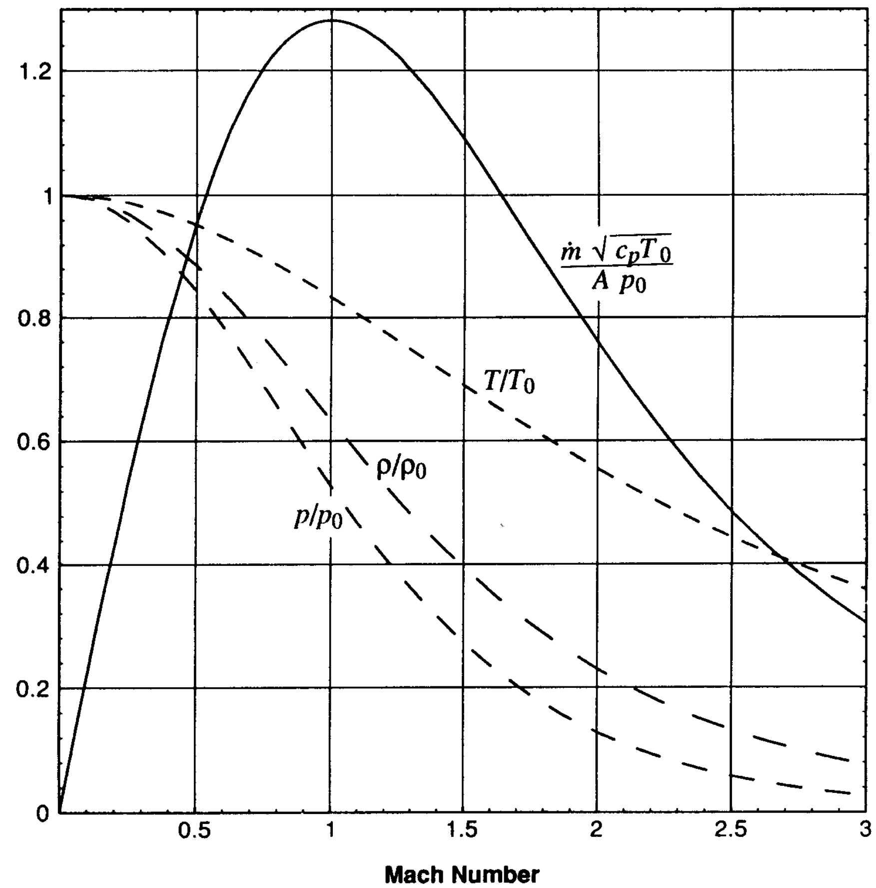


Figure 6.2. One-dimensional flow of a perfect gas, $\gamma = 1.4$