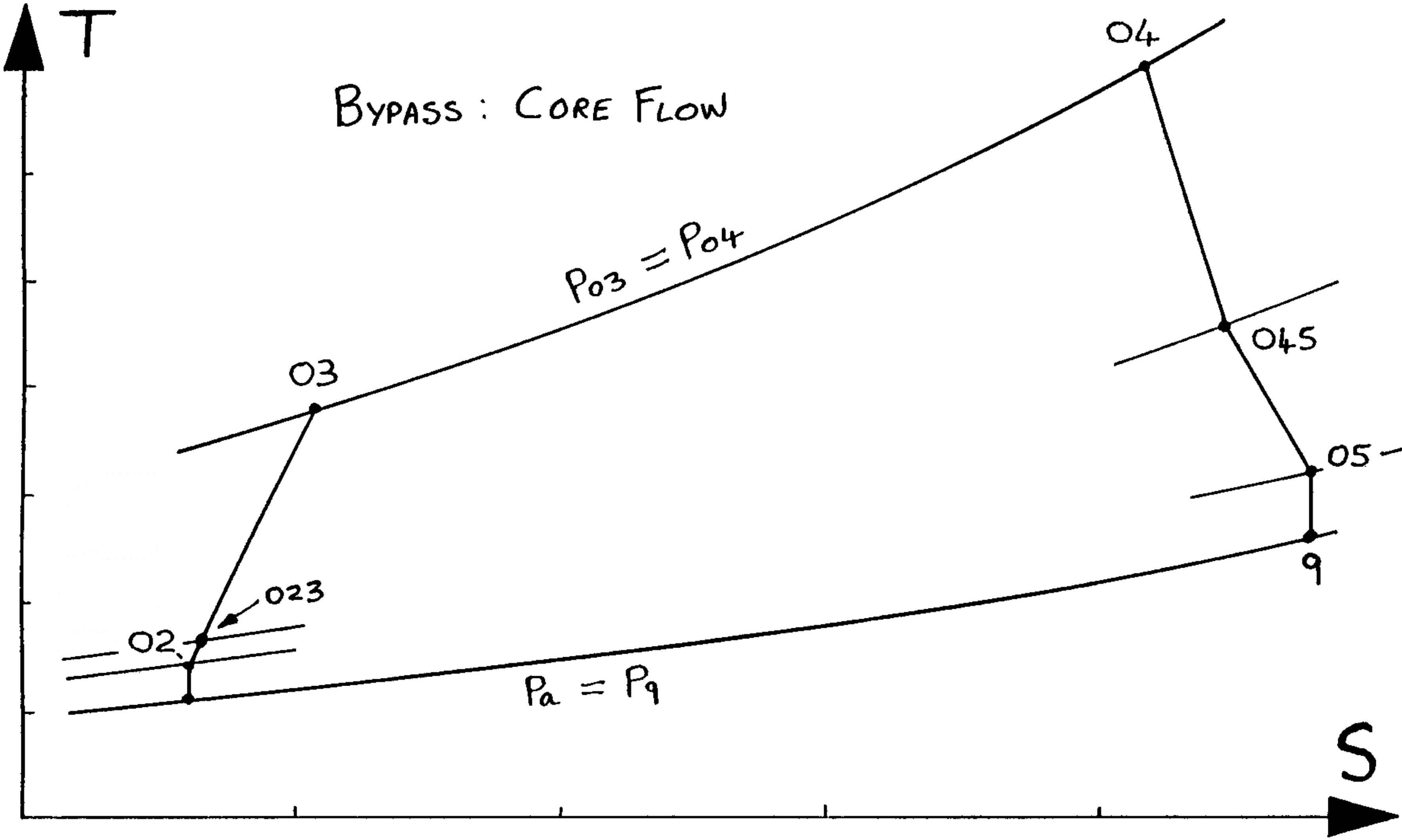
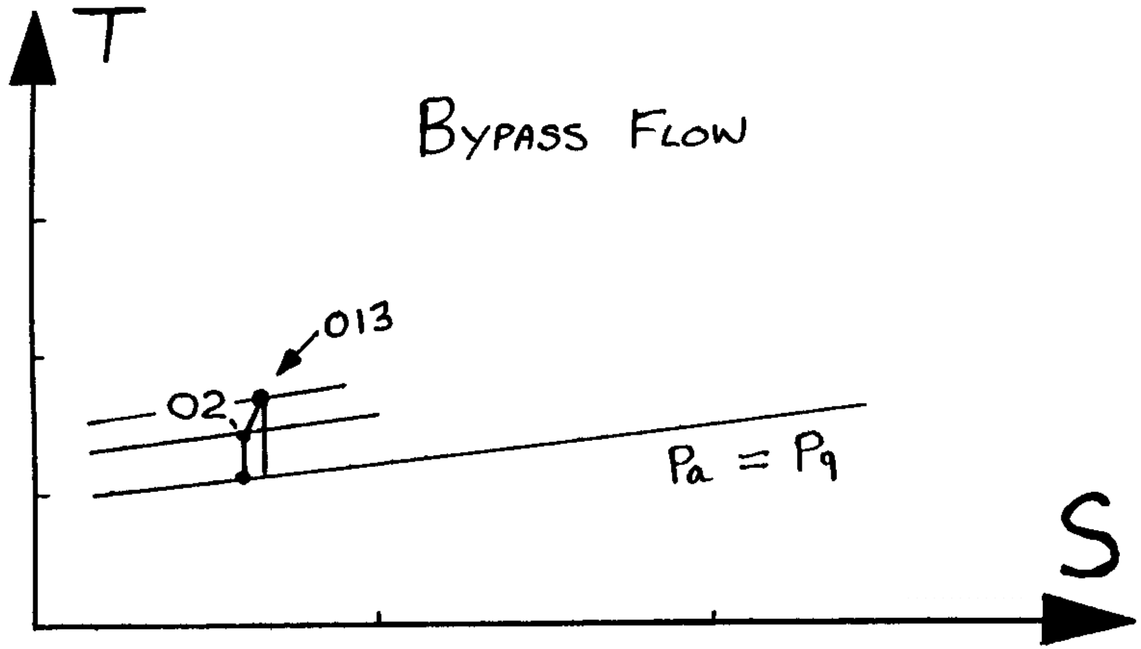
TS diagram for a high BPR engine (core-flow):



TS diagram for a high BPR engine (bypass-flow):



High bypass ratio engine flying at 31000 ft with Mach = 0.85

Standard atmosphere (31000 ft):

T_a = 227 K

p_a = 28.7 kPa

Flying at Mach = 0.85 yields:

V = 256 m/s

The stagnation conditions at engine inlet (location 02):

T_{02} = 259.5 K

p_{02} = 46.0 kPa

Fan-core flow pressure ratio of 1.6 and 90% efficient (Ex. 5.1):

Fan-core outlet conditions (location 023):

$p_{023} = p_{02} \times 1.6 = 73.6 \text{ kPa}$

$$T_{023isen} = T_{02} \times \left(\frac{p_{023}}{p_{02}}\right)^{(\gamma-1)/\gamma} = 296.8 \text{ K}$$

$T_{023} = T_{02} + (T_{023isen} - T_{02})/\eta_{fan} = 300.9 \text{ K}$

(Note: fan-core is an isentropic efficiency.)

Fan-core flow work input ($\Delta T_{0fan\text{-}core}$ = 41.4 K):

$$\dot{W}_{fan-core}/\dot{m}_c = C_p(T_{023} - T_{02}) = 41.61 \text{ kJ/kg}$$

Core compressor pressure ratio of 25 and 90% efficient (Ex. 5.1):

Compressor outlet conditions (location 03):

$$p_{03} = p_{023} \times 25 = 1840.0 \text{ kPa}$$

$$T_{03isen} = T_{023} \times \left(\frac{p_{03}}{p_{023}}\right)^{(\gamma-1)/\gamma} = 754.8 \text{ K}$$

$$T_{03} = T_{023} + (T_{03isen} - T_{023})/\eta_{comp} = 805.2 \text{ K}$$

Compressor work input (ΔT_{0core} = 504.3 K):

$$\dot{W}_{core}/\dot{m}_c = C_p(T_{03} - T_{023}) = 506.82 \text{ kJ/kg}$$

Bypass engine with turbine inlet temperature 1450 K $(T_{04}/T_{02} = 5.6)$:

Heat input required:

$\dot{m}_f LCV/\dot{m}_c = C_p(T_{04} - T_{03}) = 648.02$ kJ/kg

High-pressure turbine conditions (location 045, Ex. 5.1):

$p_{04} = p_{03} = 1840$ kPa

Shaft work energy balance:

high-pressure turbine work = core compressor work

$C_p(T_{04} - T_{045}) = \dot{W}_{HPT}/\dot{m}_c = \dot{W}_{core}/\dot{m}_c$

$T_{045} = 945.7$ K

Calculation of high-pressure turbine pressure ratio (Ex. 5.1):

$$T_{045isen} = T_{04} - (T_{04} - T_{045})/\eta_{turb} = 889.7\ \text{K}$$

$$p_{045} = p_{04} \times \left(\frac{T_{045isen}}{T_{04}}\right)^{\gamma/(\gamma-1)} = 333.0\ \text{kPa}$$

Guess required low-pressure turbine temperature drop (Ex. 7.1)

$\Delta T_{0LPT} = 361.0$ K (see page 72 for “guess”.)

Low-pressure turbine exit conditions (location 05):

$$T_{05} = T_{045} - \Delta T_{0LPT} = 584.7\ \text{K}$$

$$T_{05isen} = T_{045} - (T_{045} - T_{05})/\eta_{turb} = 544.6\ \text{K}$$

$$p_{05} = p_{045} \times \left(\frac{T_{05isen}}{T_{045}}\right)^{\gamma/(\gamma-1)} = 48.3\ \text{kPa}$$

Calculation of core jet velocity (between locations 05 and 9, Ex. 7.1):

$p_9 = p_a = 28.7$ kPa

$$T_9 = T_{05} \times \left(\frac{p_9}{p_{05}}\right)^{(\gamma-1)/\gamma} = 503.9 \text{ K}$$

$$\tfrac{1}{2} V_{jc}^2 = C_p(T_{05} - T_9)$$

$V_{jc} = 403.0$ m/s

Specification of bypass jet velocity:

$V_{jb} = V_{jc} = 403.0$ m/s

Calculation of required fan-bypass flow work input (not an isentropic efficiency):

$$C_p(T_{013} - T_{02}) = (V_{jb}^2 - V^2)/2\eta_{fan}$$

$\Delta T_{0fan\text{-}bypass} = 53.3$ K (T$_{013}$ = 312.8 K)

Power balance for low-pressure turbine shaft:

LPT work = fan-bypass work + fan-core work

$$\dot{m}_c C_p(T_{045} - T_{05}) = \dot{m}_b C_p(T_{013} - T_{02}) + \dot{m}_c C_p(T_{023} - T_{02})$$

$$\Delta T_{0LPT} = (\dot{m}_b/\dot{m}_c)\Delta T_{0fan\text{-}bypass} + \Delta T_{0fan\text{-}core}$$

Solve for the bypass ratio (Ex. 7.1):

$$BPR = \dot{m}_b/\dot{m}_c = 6.0$$

Calculation of gross thrust per unit core mass flow (Ex 7.1):

$$F_G/\dot{m}_c = (\dot{m}_c V_{jc} + \dot{m}_b V_{jb})/\dot{m}_c = 2.82 \text{ kNs/kg}$$

Calculation of net thrust (Ex. 7.1):

$$F_N/\dot{m}_c = F_G/\dot{m}_c - V(\dot{m}_c + \dot{m}_b)/\dot{m}_c = 1.02 \text{ kNs/kg}$$

Bypass engine propulsive and overall efficiency (Ex 7.1):

Propulsive efficiency:

$$\eta_p = \frac{\text{power to aircraft}}{\Delta KE} = \frac{2V}{V_j + V} = 0.777$$

(Pure turbojet gave 0.428)

Overall efficiency:

$$\eta_o = \frac{\text{Thrust} \times \text{speed}}{\dot{m}_f \, LCV} = \frac{V \times F_N / \dot{m}_c}{\dot{m}_f \, LCV / \dot{m}_c} = \frac{256 \times 1.02}{648.02} = 0.407$$

(Pure turbojet gave 0.267)

How big does the engine need to be (Ex. 7.2b):

For initial cruise at 31000 ft total thrust required (Ex. 2.3):

300.3 kN

Assuming the plane will have four engines, each engine must provide:

F_N = 75.1 kN per engine

From earlier calculation (for bypass ratio of 6):

$F_N/\dot{m}_c$ = 1.02 kNs/kg

Hence:

$\dot{m}_c$ = 73.5 kg/s

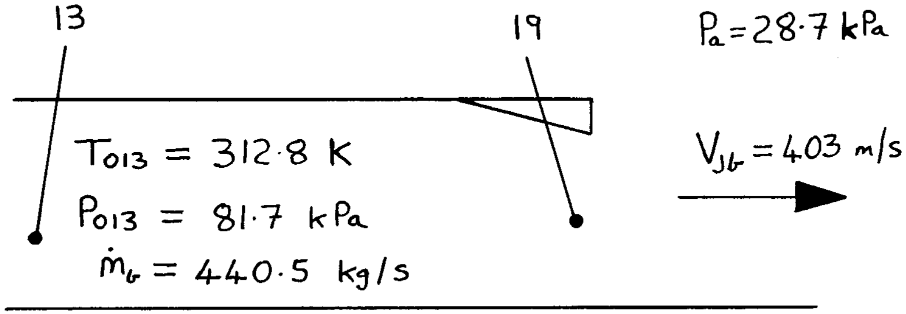
$\dot{m}_b = BPR \times \dot{m}_c$ = 440.5 kg/s

$\dot{m}_a = \dot{m}_c + \dot{m}_b$ = 514 kg/s (for **each** of four engines)

F_G = 207.3 kN per engine

Physical size of the bypass nozzle (Ex. 8.2a, page 87):

The bypass nozzle will be choked so it is possible to calculate the throat area. As it is a convergent nozzle the throat is at the exit plane, location 19.



At the choked throat the Mach number will be unity:

$$\frac{\dot{m}_b\sqrt{C_pT_{013}}}{p_{013}A_{Nb}} = 1.281$$

Hence:

$$A_{Nb} = \frac{\dot{m}_b\sqrt{C_pT_{013}}}{p_{013}\times 1.281} = \frac{440.5\times\sqrt{1005\times 312.8}}{81.7\times 10^3\times 1.281} = 2.36\ \text{m}^2$$